$y = af[b(x-h)]+k$

Assignment

1. Describe how the graph of $y = f(x)$ can be transformed to the graph of

   a) $y = f[2(x-1)] + 5$

$b=2 \quad x \to 2x$ - horizontal stretch by factor of $\frac{1}{2}$ about y-axis

$h=1 \quad x \to x-1$ - hor. translation 1 unit right + 5 units up.

$k=5 \quad y \to y-5$

   b) $y = 2f(x+4) - 5$

$a=2 \quad y \to \frac{1}{2}y$ - vertical stretch by a factor of 2 about x-axis,

$h=-4 \quad x \to x+4$ - translation 4 units left and 5 units down.

$k=-5 \quad y \to y+5$

   c) $y = f\left(\frac{1}{2}x + 6\right) + 1$

$b = \frac{1}{2} \quad x \to \frac{1}{2}x$ - horizontal stretch by a factor of 2 about the y-axis,

$h=-6 \quad x \to x+6$ - translation 6 units left and 1 unit up.

$k=1 \quad y \to y-1$

2. Consider the function $y = f(x)$. In each case determine:
   - the replacements for x and y which would result in the following combinations of transformations
   - the equation of the transformed function in the form $y = af[b(x-h)] + k$

   a) a horizontal stretch by a factor of 3 about the y-axis and a vertical translation of 6 units up.

$x \to \frac{1}{3}x \qquad y-6 = f\left(\frac{1}{3}x\right)$

$y \to y-6 \qquad y = f\left(\frac{1}{3}x\right) + 6$

   b) a reflection in the y-axis, a horizontal translation of 3 units right, and a vertical translation of 5 units down.

$x \to -x \qquad y = f(-x)$

$x \to x-3 \qquad y = f(-(x-3)) \qquad y = f(-x+3)$

$y \to y+5 \qquad y+5 = f(-x+3) \qquad y = f(-x+3) - 5$

   c) a horizontal stretch by a factor of $\frac{2}{3}$ about the y-axis, a vertical stretch by a factor of $\frac{2}{5}$ about the x-axis, a reflection in the x-axis, and a vertical translation of 1 unit up.

$x \to \frac{3}{2}x \qquad y = f\left(\frac{3}{2}x\right)$

$y \to \frac{5}{2}y \qquad y = \frac{2}{5}f\left(\frac{3}{2}x\right)$

$y \to -y \qquad y = -\frac{2}{5}f\left(\frac{3}{2}x\right)$

$y \to y-1 \qquad y = -\frac{2}{5}f\left(\frac{3}{2}x\right) + 1$

3. Describe how the graph of the second function compares to the graph of the first function.

a) $y = x^4$, $-4y = (x-2)^4$

$y \to -4y$ - vert. stretch by factor of $\frac{1}{4}$ about x-axis + reflection in x-axis.

$x \to x-2$ - translation 2 units right.

b) $y = |x|$, $y = |\frac{1}{3}(x+2)|$

$x \to \frac{1}{3}x$ - horizontal stretch by a factor of 3. + horizontal

$x \to x+2$ translation 2 units ~~rig~~ left.

c) $y = \sqrt{x}$, $y - 1 = 2\sqrt{4x-8}$

$y \to \frac{1}{2}y$

$y \to y-1$ - vertical stretch by a factor of 2

$x \to 4x$ - horizontal stretch by a factor of $\frac{1}{4}$, translation

$x \to x-2$ 2 units right + 1 unit up.

4. In each case the transformations are applied in the order given to transform the graph of $y = f(x)$ to the graph of $y = af[b(x-h)] + k$. Determine the values of $a, b, h,$ and k.

a) a horizontal stretch by a factor of $\frac{3}{5}$ about the y-axis and a reflection in the x-axis

$x \to \frac{5}{3}x$ $y = -f(\frac{5}{3}x)$ $a = -1$ $h = 0$

$y \to -y$ $b = \frac{5}{3}$ $k = 0$

b) a vertical stretch by a factor of $\frac{1}{3}$ about the x-axis and a reflection in the y-axis

$y \to 3y$ $y = \frac{1}{3}f(-x)$ $a = \frac{1}{3}$ $h = 0$

$x \to -x$ $b = -1$ $k = 0$

c) a vertical stretch by a factor of 2 about the x-axis, then a translation 5 units to the left and 2 units up

$y \to \frac{1}{2}y$ $y \to y-2$ $y = 2f(x+5) + 2$ $a = 2$ $k = 2$.

$x \to x+5$ $b = 1$

$h = -5$

d) a horizontal stretch by a factor of 4 about the y-axis, a vertical stretch by a factor of 2 about the x-axis, a reflection in the y-axis and then a translation of 10 units down

$x \to \frac{1}{4}x$ ~~y~~ $y = 2f(\frac{1}{4}x)$ $x \to -x$ $y = 2f(-\frac{1}{4}x) - 10$ $a = 2$ $h = 0$

$y \to \frac{1}{2}y$ $y \to y+10$ $b = -\frac{1}{4}$ $k = -10$

e) a translation of 6 units right, then a horizontal stretch by a factor of $\frac{1}{2}$ about the y-axis and a reflection in the x-axis

$x \to x-6$ $y = f(x-6)$ $a = -1$

$x \to 2x$ $y = f(2x-6)$ $b = 2$

$y \to -y$ ~~$y = -f(2x-6)$ $h = 3$~~

$y = -f(2(x-3))$ $k = 0$

5. The function $f(x) = \sqrt{x}$ is transformed into the function $g(x)$ by stretching horizontally by a factor of 6 about the y-axis, stretching vertically by a factor of 3 about the x-axis, reflecting in the x-axis, and translating 1 unit up and $\frac{1}{2}$ unit to the right. Write the equation for $g(x)$

$x \to \frac{1}{6}x$, $y \to \frac{1}{3}y$ $\Big\}$ $y = 3\sqrt{\frac{1}{6}x}$

$y \to -y$ $\quad y = -3\sqrt{\frac{1}{6}x}$

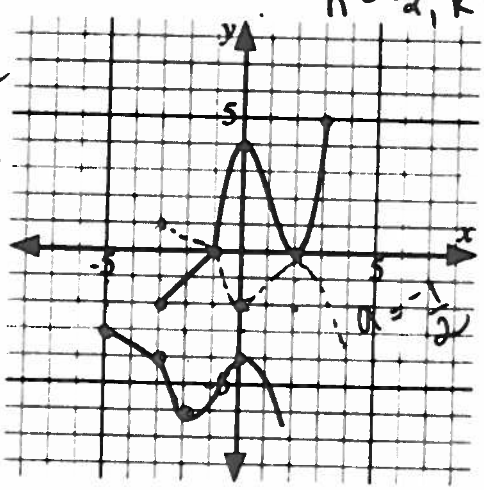
$x \to x - \frac{1}{2}$, $y \to y - 1$ $\Big\}$ $y - 1 = -3\sqrt{\frac{1}{6}(x-\frac{1}{2})}$ $\quad y = -3\sqrt{\frac{1}{6}(x-\frac{1}{2})} + 1$

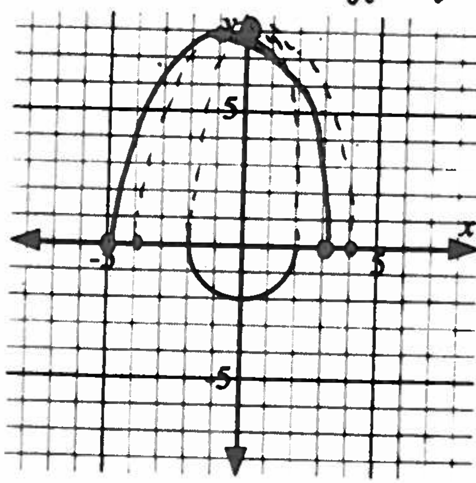
6. The graph of $y = f(x)$ is shown. Sketch the graph of:

a) $y + 4 = -\frac{1}{2}f(x + 2)$ $\quad a = -\frac{1}{2}$ $\quad h = -2,\ k = -4$

b) $y = -4f\left(\frac{1}{2}x + 1\right)$ $\quad y = -4f\left(\frac{1}{2}(x+2)\right)$ $\quad a = -4 \quad b = \frac{1}{2} \quad h = -2$

- ver stretch + reflect of $\frac{1}{2}$
$\leftarrow 2 \cdot + \downarrow 4$





vert stretch + reflect by -4.
- horz stretch by 2
- hoz. trans 1 u L

$y \to 2y$, $y \to -y$ $\Big\}$ vert strech + reflect

$x \to x+2$, $y \to y+4$ $\Big\}$ 2 u. L, 4 u $\downarrow$

$y \to \frac{1}{4}y$, $y \to -y$ $\Big\}$ vert. stretch + (x4) reflect

$x \to \frac{1}{2}x$ $\quad$ hor. stretch x 2

$x \to x+2$ $\quad$ 2 u Left.

7. The function $f(x) = \sin x°$ is transformed into the function $g(x)$ by stretching horizontally by a factor of $\frac{1}{4}$ about the y-axis, stretching vertically by a factor of $\frac{2}{3}$ about the x-axis, reflecting in the y-axis, and translating 5 units down. Write the equation for $g(x)$.

$x \to 4x$ $\quad y = \sin 4x°$

$y \to \frac{3}{2}y$ $\quad y = \frac{2}{3}\sin 4x°$

$x \to -x$ $\quad y = \frac{2}{3}\sin(-4x)°$

$y \to y+5$ $\quad y = \frac{2}{3}\sin(-4x)° - 5$

$g(x) = \frac{2}{3}\sin(-4x)° - 5$

8. The function $f(x) = \frac{1}{x}$ is transformed into the function $g(x)$ by stretching horizontally by a factor of 2 about the y-axis, stretching vertically by a factor of 5 about the x-axis, and translating 3 units to the left. Write the equation for $g(x)$.

$x \to \frac{1}{2}x$
$y \to \frac{1}{5}y$
$x \to x+3$

$y = \frac{1}{\frac{1}{2}x}$ $y = \frac{2}{x}$

$1 \div \frac{1}{2}x = 1 \cdot \frac{2}{x} = \frac{2}{x}$

$\frac{1}{5}y = \frac{2}{x}$ $y = \frac{10}{x}$

$y = \frac{10}{x+3}$ $g(x) = \frac{10}{x+3}$

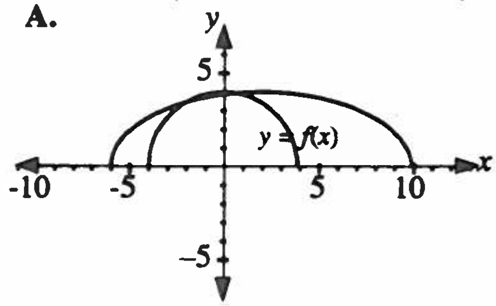
Multiple Choice **9.** The graph of $y = f(x)$ is the semi-circle centred at the origin. Which of the following shows the graph of $y = f(x)$ and $y = f(2x - 4)$?

$x \to 2x$
$x \to x-2$

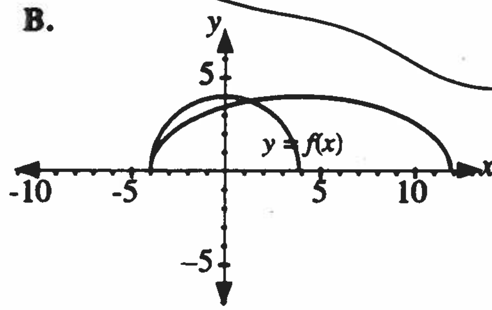
$y = f(2(x-2))$

$x \div 2$, 2U(R)

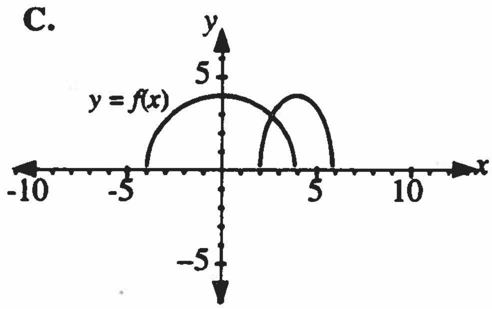
A.



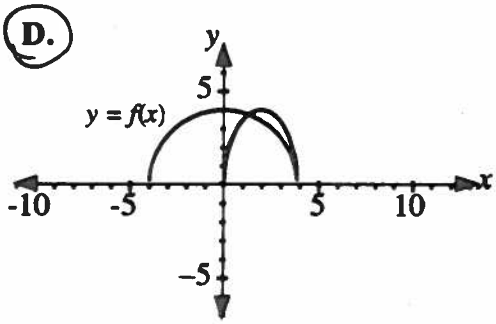
B.



C.



D.



Numerical Response **10.** The graph of $y = \frac{1}{x}$ is transformed to the graph of $y = \frac{1}{5(x-3)} + 4$ by a series of transformations. One of these transformations is a vertical stretch about the x-axis. The scale factor of the vertical stretch, to the nearest tenth, is ______ .

(Record your answer in the numerical response box from left to right.)

0	.	2	

$y \to 5y$
$x \to x-3$
$y \to y-4$

vertical stretch factor $= \frac{1}{5} = 0.2$

11. The points $P(2, 4)$ and $Q(4, 2)$ are on the graph of $y = f(x)$. If the function f is transformed to function g, where $g(x) = 2f(2x) - 1$, then the points P and Q are transformed to $R(a, b)$ and $S(c, d)$, respectively.

Write the value of a in the first box. $(2,4) \to (2,8) \to (1,8) \to (\underline{1},\underline{7})$
Write the value of b in the second box. $(4,2) \to (4,4) \to (2,4) \to (\underline{2},\underline{3})$
Write the value of c in the third box.
Write the value of d in the fourth box.

(Record your answer in the numerical response box from left to right.) | 1 | 7 | 2 | 3 |

$y = f(x)$
$y \to \frac{1}{2}y$ $\quad y = 2f(x)$ vert. stretch factor 2 $(y \times 2)$
$x \to 2x$ $\quad y = 2f(2x)$ hori. stretch about y axis - factor $\frac{1}{2}$ $(x \div 2)$
$y \to y+1$ $\quad y = 2f(2x) - 1$ translation 1 unit down $(y - 1)$

Answer Key

1. a) horizontal stretch by a factor of $\frac{1}{2}$ about the y-axis, then a translation 1 unit right and 5 units up
b) vertical stretch by a factor of 2 about the x-axis, then a translation 4 units left and 5 units down
c) horizontal stretch by a factor of 2 about the y-axis, then a translation 12 units left and 1 unit up

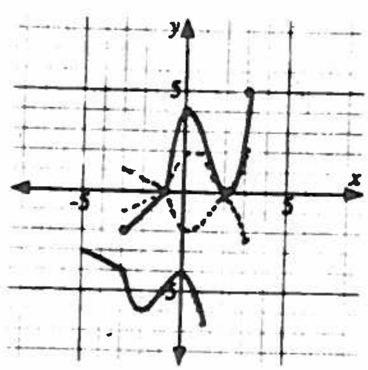
2. a) replace x with $\frac{1}{3}x$ and y with $y - 6$ $\quad y = f\left(\frac{1}{3}x\right) + 6$
b) replace x with $-x$, x with $x - 3$, and y with $y + 5$ $\quad y = f(-(x - 3)) - 5$ or $y = f(-x + 3) - 5$
c) replace x with $\frac{3}{2}x$, y with $\frac{5}{2}y$, y with $-y$, and y with $y - 1$ $\quad y = -\frac{2}{5}f\left(\frac{3}{2}x\right) + 1$

3. a) vertical stretch by a factor of $\frac{1}{4}$ about the x-axis, reflection in the x-axis, and a horizontal translation 2 units right
b) horizontal stretch by a factor of 3 about the y-axis, then a horizontal translation 2 units left
c) vertical stretch by a factor of 2 about the x-axis, horizontal stretch by a factor of $\frac{1}{4}$ about the y-axis, then a translation 2 units right and 1 unit up

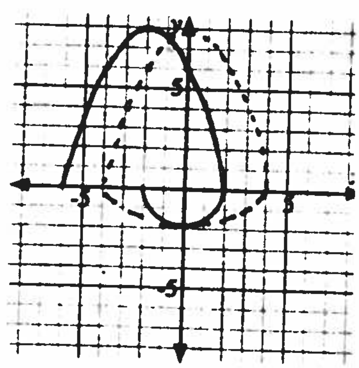
4. a) $a = -1$ $\quad b = \frac{5}{3}$ $\quad h = 0$ $\quad k = 0$ $\qquad$ b) $a = \frac{1}{3}$ $\quad b = -1$ $\quad h = 0$ $\quad k = 0$
c) $a = 2$ $\quad b = 1$ $\quad h = -5$ $\quad k = 2$ $\qquad$ d) $a = 2$ $\quad b = -\frac{1}{4}$ $\quad h = 0$ $\quad k = -10$
e) $a = -1$ $\quad b = 2$ $\quad h = 3$ $\quad k = 0$

5. $g(x) = -3\sqrt{\frac{1}{6}\left(x - \frac{1}{2}\right)} + 1$

6. a)



b)



7. $g(x) = \frac{2}{3}\sin(-4x°) - 5$

8. $g(x) = \dfrac{10}{x + 3}$

9. D

10. | 0 | . | 2 | |

11. | 1 | 7 | 2 | 3 |