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Mach's Principle: Why Are Some Reference Frames Inertial, And Others Not?

Joseph Walbran

INTRODUCTION

When you take a physics class, one of the first things you learn is that a laboratory on the second floor of a building is exactly the same as a laboratory on a moving passenger train. From inside, there's no way to differentiate the two laboratories—if the train car has no windows, there's no experiment you can do that can detect the train's velocity. (We're imagining that the train has a very-high-quality suspension system, so you don't feel any jittering from the tracks.) In particular, Newton's law of inertia holds in both laboratories. Any object in motion will travel in a straight line at a constant velocity, unless acted upon by a force; this is just as true on a moving train as it is in a solid brick building (Einstein 111).

But a laboratory on a train that is just leaving the station is very different. Even if there are no windows in your train car, you can feel that the train is accelerating—the ground moves out from under you, and you fall backwards towards the rear of the car. Newton's law of inertia does *not* hold in this laboratory. If you roll a ball bearing along the ground while the train is leaving the station, it won't travel in a straight line: it will curve toward the back of the car.

We say that each laboratory defines a *reference frame*—that is, a set of points we consider to be stationary for the duration of the experiment. All motion is measured relative to the points in the reference frame. The first two laboratories—the one on the building, and the one on a train moving at a constant velocity—are said to be *inertial* reference frames, since if we consider those laboratories to be stationary, we find that Newton's law of inertia holds. The laboratory on the accelerating train is *not* an inertial reference frame, since if we consider that laboratory to be stationary, Newton's law of inertia does not describe the motion of objects.

You might wonder why some reference frames are inertial and others aren't. What's special about the first two laboratories that makes objects move in a straight line? Why don't objects move in a straight line in the third laboratory?

The usual answer is that the third laboratory cannot be used as an inertial reference frame because it's accelerating. The first two laboratories are not accelerating; they're moving at a constant velocity. In any laboratory moving at a constant velocity, objects travel in a straight line, so those reference frames are inertial.

This is the answer that people usually give, but it's not very satisfying. When we say the third laboratory is accelerating, what does that mean? We might just as well argue that laboratory three is stationary, and *the Earth* is accelerating relative to *it* (Einstein 112-113).

We need some standard for determining which objects are accelerating and which objects aren't. What is acceleration measured against? The Earth? The Sun? The center of the galaxy? This question troubled physicists around the turn of the 20th century. Newton had said that acceleration is measured relative to some sort of absolute space, which he believed was a real, physical thing (Rindler 4-5). But the notion of absolute space never sat particularly well with physicists and philosophers. They were particularly disturbed that Newton had only seemed to believe in absolute space because it was expedient, not because there was any hard evidence for its existence. The physicist Ernst Mach, writing a textbook on mechanics in 1893, has this to say:

> It is scarcely necessary to remark that in the reflections here presented Newton has again acted contrary to his intention only to investigate *actual facts*. No one is competent to predicate things about absolute space and absolute motion; they are pure things of thought, pure mental constructs, that cannot be produced in experience. (229)

Mach's principle—first articulated by Ernst Mach in the same textbook—is an attempt to explain which reference frames are inertial in a way that does not depend on the existence of absolute space. Today, Mach's principle is not widely accepted, but it has not been completely refuted, either (Weinberg 87-88; Rindler 11, 14, 242-244). The principle enjoyed brief popularity as a theory of inertia in the 1910s and 1920s, when it was championed by Albert Einstein. It was an important inspiration for general relativity (GR), and Einstein tried hard to incorporate Mach's principle into GR before eventually giving up the pursuit (Norton 10-11; Sciama 35).

MACH'S PRINCIPLE

According to Mach's principle, large accelerating masses induce a local inertial reference frame around them (Rindler 10). Let's go back to the case of laboratory three, the train that's departing from the station. Imagine that, as the train is departing, an incredibly large mass passes through space overhead. This mass is moving in such a way that it exactly follows the motion of the train on the ground. (We don't concern ourselves with why the mass follows that particular trajectory. We just assume that there's a good reason.) Then, according to Mach's principle, the large mass in the sky becomes the local standard of non-acceleration. Since laboratory three is not accelerating relative to the mass in the sky, it becomes an inertial reference frame. In the train leaving the station, objects move in a straight line. But every building on Earth *is* accelerating relative to the mass in the sky, so they are *not* inertial reference frames. Everywhere else on earth, a thrown object will curve in the air.

Another example will be helpful. Imagine you're outside on a warm summer night. The ground underneath your feet is spinning around you, as are the stars above your head. You notice that your arms are pulled outwards. Then, the ground and the stars come to a stop. They are not spinning around you anymore. You notice that your arms lie flat at your sides. How do we explain this behavior? Steven Weinberg remarks on this experiment,

> It would surely be a remarkable coincidence if the inertial frame, in which your arms hung freely, just happened to be the reference frame in which typical stars are at rest, unless there were some interaction between the stars and you that determined your inertial frame. (17)

Mach's principle says that there *is* such an interaction. The centrifugal force that pulls your arms outwards is directly caused by the distant stars that are spinning around you. When these large masses spin, they create a local inertial reference frame, and since you're accelerating relative to that reference frame, you experience centrifugal forces (Rindler 12).

For Mach, the presence of any large rotating mass would be sufficient to cause a centrifugal force. Suppose, instead of the stars spinning around you, you sat in a room with very thick walls made of a very dense metal. Then, we rotate the walls of the room around you at a very high speed. If Mach's principle is true, your arms would be pulled outwards by a centrifugal force. The spinning cylindrical shell would induce a local inertial reference frame, just like the stars did in the first case; and, since you would be accelerating relative to the cylindrical shell, you would feel a centrifugal force (Rindler 12-13). So far, no experiment has been able to verify this effect. But that doesn't mean the effect doesn't exist; it may just require a very large mass to be spinning before any forces are detectable (Rindler 14).

MAXWELLIAN THEORIES OF GRAVITY

Mach's proposal, that a rotating ring of mass will induce forces on objects in the center, is not entirely surprising. It resembles electromagnetic induction, the phenomenon whereby a rotating ring of charged particles will induce a magnetic field in its center (Rindler 11). The laws governing electromagnetic induction were codified by James Clerk Maxwell in the late 1800s, just a few decades before Mach introduced his principle, so we would expect Mach and his followers to have taken direct inspiration from Maxwell's work. With Mach's principle, they might have been trying to create a theory of gravity analogous to Maxwell's laws of electromagnetism.

Surprisingly, this does *not* seem to have been the case. John D. Norton notes that, while there was some interest in Maxwellian theories of gravity around the turn of the 20th century, Mach and his followers barely mention these theories except in the occasional footnote. Renn, one of Norton's colleagues, suspects that this was because of siloing within the physics community. Back in the late 1800s, scientists like Mach, who studied Newtonian mechanics, rarely compared notes with scientists like Maxwell, who studied electromagnetism (Norton 57).

The separation between the mechanics community and the electromagnetism community started to break down in the 1900s; this is especially due to the work of Einstein, who was active in both disciplines (Norton 57). And, as lines of communication started to open, more scientists started to see the similarities between electromagnetic induction and the inductive effects of Mach's principle.

Research into gravity has been dominated by general relativity ever since Einstein devised GR in the 1910s. Still, though, Maxwellian theories of gravity have been intermittently popular throughout the twentieth century; they are particularly associated with the work of D. W. Sciama, a proponent of Mach's principle (Rindler 11). In his 1953 paper, Sciama laid out a mathematical theory of gravity with Maxwellian characteristics that includes Mach's principle—that is to say, in Sciama's system, accelerating masses create local inertial frames around them (Sciama 34). Sciama's system is intended as a proof-of-concept, rather than a complete theory of gravity. For example, he makes quite a few simplifying assumptions in order to make the math cleaner (Sciama 34). This means that, unfortunately, his system is not good at making predictions about the world; it only serves as a preview of what a full Maxwellian theory of gravity might look like.

Unfortunately, besides just being a simplified model, there are also some serious foundational issues with Sciama's theory of gravity. Sciama himself notes that his system departs from general relativity in a few places, but Rindler remarks that the system isn't even compatible with *special* relativity (SR). This is a problem that Maxwellian theories of gravity often struggle with: under SR, the analogy between gravity and electromagnetism starts to fall apart. Usually, in this analogy, mass is treated as the analogue of electric charge—the former sets up a gravitational field in the same way the latter sets up an electric field. But mass and charge behave very differently under SR. In SR, charge is conserved: the total amount of charge in a system is always constant. But mass is decidedly *not* conserved: under SR, an object has more mass when it's in motion than it has when it's sitting still. Because mass is a quantity that fluctuates in SR, Maxwell-style laws tend not to describe gravity very well (Sciama 34; Rindler 11).

GENERAL RELATIVITY AND THE EQUIVALENCE PRINCIPLE

We've danced around general relativity for a while, but now we should discuss it explicitly. Today, GR is mostly remembered as a theory of gravity—its key insight being that a gravitational field is just a curved region of spacetime. However, when Einstein introduced GR in 1916, he did not present it as a theory of gravity, but rather, as an answer to the question "why are some reference frames inertial and others not?"—the same question that Mach had been trying to answer with his principle (Rindler 10, 20-21; Einstein 109-112).

Einstein's answer to this question is similar to Mach's. In GR, as in Mach's principle, large masses create local inertial reference frames around them. But GR and Mach's principle differ in how these reference frames are created.

To get a sense for how reference frames in GR work, consider the following thought experiment. Imagine you're in the cabin of an airplane in free-fall. The windows have been painted over, so you can't see outside. You throw a tennis ball. You will see the tennis ball move in a straight line, just like Newton says it would move in an inertial reference frame.

Now, consider instead the following scenario. You are in the same airplane, but instead of falling through the atmosphere, the airplane is floating in the void of interstellar space. There is very little gravity here, so objects around you float freely. You throw a tennis ball. It moves in a straight line, as it is expected to move in an inertial reference frame.

The key insight of GR is that, if the windows of the airplane are covered, these two scenarios are indistinguishable. There is no experiment you can do that can decide whether a laboratory is in free-fall above a planet, or whether it is floating in interstellar space. This is called the *equivalence principle* (or the EP, for short) and it is our starting point for understanding inertial reference frames (Rindler 17).

Neither Newton nor Mach would call the free-falling airplane an inertial reference frame. Instead, they would have said something like, "The reference frame of the airplane is accelerating with regard to the distant stars, so it is not inertial. However, the Earth's gravitational field causes all objects inside the cabin to assume the same acceleration as the airplane itself, and so it *appears*, to the passengers, as if the airplane were not accelerating." But this is not how Einstein analyzes the situation. In Einstein's view, the airplane in free-fall appears to be an inertial reference frame, so it is an inertial reference frame. But he goes a step further: a laboratory inside the cabin would be unable to detect any gravitational field. So, in the airplane's reference frame, the gravitational field doesn't exist. It's been canceled out by the airplane's acceleration (Rindler 17-19).

We can explore the relationship between gravitational fields and reference frames further. Suppose we have a rocket ship deep in interstellar space, with a number of astronauts onboard. The voyage would be much more pleasant if there were gravity within the rocket, so that the astronauts could stick to the floors, and place objects on tables without them drifting away. In science fiction, a common solution to this problem is to give the rocket ship "artificial gravity" by means of acceleration. If the rocket fires its thrusters, causing it to accelerate, then the floor of the ship will press upwards against the astronauts, giving the impression that there is gravity within the ship.

Newton would say that, since the rocket is accelerating relative to the distant stars, it is no longer an inertial reference frame. Because it isn't an inertial frame, objects will move in curved paths, which is why it looks like there's gravity inside the ship.

Mach would say that, since the distant stars are accelerating relative to the rocket, they exert a force on the rocket and on the objects inside; that force from the distant stars pushes the astronauts onto the floor of the rocket. This is why it looks like there's gravity inside the rocket ship (Rindler 18).

Einstein, by contrast, would say that there *really is* a gravitational field inside the ship. When the rocket changes its reference frame—from a frame that is a free-floating to a frame that is accelerating—a gravitational field appears where there was none before (Einstein 114; Rindler 18).

In general relativity, then, there isn't really a distinction between inertial frames and non-inertial frames: what we would call a non-inertial frame is just an inertial frame with a gravitational field overlayed. In his 1916 paper introducing GR, Einstein writes:

> Of all imaginable spaces R_1, R_2, etc., in any kind of motion relatively to one another, there is none which we may look upon as privileged *a priori* without reviving the above-mentioned epistemological objection. *The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion.* (113)

GENERAL RELATIVITY AND MACH'S PRINCIPLE

The laws which govern the relationship between reference frames and gravitational fields are incredibly complicated, much more so than Maxwell's laws of electrodynamics (Rindler 11). Because of GR's complexity, it's not entirely clear how GR relates to Mach's principle. Mach's principle, recall, says that accelerating masses will exert a force on other nearby masses; and while GR sometimes predicts similar effects, it is unclear if the consequences of GR exactly match the consequences of a hypothetical Mach force. The question is especially fuzzy because

Mach's principle was only ever a qualitative statement, and never had a mathematical formulation (Rindler 11).

With that being said, GR does directly contradict some of the philosophy that led Mach to formulate his principle. In particular, Mach objected to the idea that space was a physical entity, since it didn't seem possible to observe or influence space in any way. This runs counter to the way GR works. In GR, space is absolutely a physical thing, and its curvature can be both observed and influenced by the presence of masses (Rindler 242-243).

Moreover, Mach thought that inertial reference frames are uniquely determined by the position and motion of all masses in the universe. (That is, if you're standing at a given point, and you know the trajectory of every object in the universe, you can predict which reference frames are going to be inertial at the point where you're standing.)

It turns out that this is *not* the case in GR. In GR, we say that the inertial frames at a point are those frames which set the gravitational field at that point to zero. So, to figure out which frames are inertial, we just need to figure out how gravity acts at a point. As it happens, you cannot determine how gravity acts as a point just by knowing the mass and trajectory of every object in the universe. The gravitational field at a point also depends on certain cosmological properties, which can affect the curvature of space. Since Mach doesn't want to consider the existence of space, Mach's principle can't account for these cosmological factors (Rindler 243).

CONCLUSION

Mach's principle states that large accelerating masses will induce local inertial reference frames around them. This principle has not been entirely abandoned today; it still crops up in some physical theories, such as those proposed by Sciama. But the philosophical motivations behind Mach's principle—namely, skepticism about the existence of space—are now mostly non-issues, and general relativity has largely superseded Mach's principle as a way to explain inertial reference frames. The principle, then, is interesting mostly as a false-start in the history of worldviews. It almost never achieved mainstream acceptance, except for a brief period in the 1910s and 1920s. Despite this, it had a disproportionate impact on the worldviews that came after it. Though few scientists believe in Mach's principle, many scientists owe something to it.

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